# Reflection and Transmission of Plane Waves Incident at the Interface between an Elastic Basement and a Sedimentary Layer Underlying a Homogeneous Ocean

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## Abstract

An acoustical model for consolidated sediments is used to study the reflection in the ocean sediments. The problem of reflection and refraction of elastic waves incident at the interface between an elastic basement and a sedimentary layer lying under a uniform layer of liquid is studied. The sedimentary layer has been taken as transversely isotropic liquid-saturated porous solid. The interface between the sedimentary layer and the elastic half-space is taken as imperfect interface and appropriate boundary conditions are defined there at. The assumptions made are justified at most frequencies of practical interest in underwater acoustics. The reflection and refraction coefficients have been obtained and studied for different degrees of bonding of the sediments-basement interface for all angle of incidence.



## Introduction

The bottoms of ocean are usually covered with a variety of sediments. The influence of the bottom sediment on the propagation of acoustic, gravity, or seismic waves becomes important when water depth is comparable to the wavelength of the waves. Sometimes, the sediment is considered as a fluid or an elastic solid with a small rigidity but the mechanics of marine sediment is complicated because sediment is coupled multiphase medium: the solid phase of grain and the fluid phase of the pore water and gases. It is further complicated when effects of imperfect elasticity of the skeletal frame are also considered. The problem of describing seismic/elastic waves propagating in the ocean over a porous sea bed is of interest for geophysicist, engineers and acousticians for a variety of reasons. In some cases, interest is focused on low-frequency waves of large amplitude e.g. those waves which arise near the source of an earthquake. At other times, the main interest is in waves of low frequency and amplitude that have traveled long distances through the sediment. In another category, high frequency waves that are able to resolve thin layering and other fine structural details are of interest in studying bottom topography and near bottom sediments.

Biot (1941, 1956a, 1956b, 1962a, 1962b) published a series of papers on the theory of elastic wave propagation in saturated sediments. The theory suggests that two kind of dilatational waves propagate in a fluid filled porous medium with inter connected voids. The wave of the “first kind” is similar to the usual dilatational waves in an ordinary elastic body except that there is small amount of dispersion and some attenuation that depends on frequency. On the other hand, the wave of the

"second kind" is a low-velocity, highly attenuated wave with marked dispersion. Very little energy is converted into waves of the "second kind" in water-saturated sediment because of the relatively low compressibility of the pore water compared with the skeletal frame. Existence of slow compressional waves at frequencies much higher than the relaxation frequency was observed in the fused glass beads by Plona (1980). Chotiros (1995) has also claimed to detect the "second kind" of dilatational waves which propagate with a velocity near to 1200m/s in the bottom sand.

Different studies have been made on the wave propagation in porous media on the basis of Biot's theories. A mathematical model, based on Biot theory, that takes into account both ingranular losses and viscous losses in the interstitial fluid of the saturated sediments has been proposed by Stoll and Bryan (1970) and used by Stoll (1974, 1977, 1979, 1980) in his different studies on propagation and attenuation of acoustic waves in sediments. Stoll and Kan (1981) studied the reflection of plane acoustic waves at a water-sediment interface. Many problems on acoustic waves in saturated or unsaturated sediments have been studied on the basis of the Biot theory. The effects of the sediment properties on the dispersion and the attenuation of acoustic waves were examined by Yamamoto (1983) in the study of acoustic normal modes in a homogeneous ocean overlying a homogeneous porous elastic bottom half-space. Collins et al. (1997) investigated that some ocean sediments may be modeled as poroelastic media with relatively high speed of slow dilatational wave and relatively low speed of shear wave. The scattering of acoustic waves from a gassy poroelastic seabed was studied by Boyle and Chotiros (1998). The asymptotic formulae for attenuation coefficients and sound speed in ocean sediments were given by Badiey et al. (1998) for high, low and intermediate frequencies. Buckingham (2000) developed a linear theory of wave propagation in saturated, unconsolidated granular materials including marine sediments. Two type of shearing, translational and radial, which occur at grain contacts during the passage of a wave, were considered in this study of stress-relaxation mechanism in saturated, unconsolidated marine sediments. Liu et al. (2001) studied the effects of acoustic properties of seabed, including the density and sound speed of the sedimentary layer and sub-bottom, on the wave field characteristics.

Many researchers studied the reflection and transmission phenomena in fluid saturated elastic solids. The reflection loss during acoustic wave scattering from solid boundaries at the ocean bottom was calculated by Kuo (1992). The mode conversions during the reflection and transmission of seismic waves at the boundaries of porous media were studied by de la Cruz et al. (1992). Badiey et al. (1994) studied the wave reflection from inhomogeneous anisotropic poroelstic seafloor by using propagator matrix method. The reflection and refraction at a plane interface between fluid and non dissipative porous solid was studied by Sharma (2004b). Lin et al. (2006) discussed the surface displacements, surface strain, rocking and energy partitioning during the reflection of waves in a fluid saturated poroelastic half-space. Recently, Chotiras et al. (2007) studied the refraction and scattering into sandy ocean sediments and observed the sound

penetration from acoustic wide band array buried into a sandy sediment.

The fluid saturated geological materials show anisotropy in their elastic properties and permeability due to bedding, compaction and presence of aligned microcracks. Sharma and Gogna (1991) studied the propagation of Rayleigh waves on the surface of transversely isotropic liquid saturated porous layered medium. Wave propagation in an anisotropic periodically multilayered medium was discussed by Potel et al. (1993). Three dimensional mathematical models for propagation of four quasi-waves in general anisotropic fluid saturated porous solids were developed by Sharma (2004a, 2005). Vashishth and Khurana (2005) analyzed the Rayleigh modes in transversely isotropic heterogeneous poroelastic layers.

In order to model sedimentary bottom in a realistic way, the anisotropy of their physical properties should be taken into account. In this paper, we study the reflection and refraction of seismic waves at a boundary between elastic half-space and porous layer which is underlying a liquid layer. This model corresponds to homogeneous ocean with a poroelastic bed overlaying the elastic basement. The sedimentary layer is considered as transversely isotropic. The problem stated above has not been studied so far and this is of interest not only for theoretical work but also for practical applications. Layers of porous solids, such as sandstone or limestone etc., saturated with oil or ground water are present in the oceanic crust and are of interest in geophysical exploration

## Formulation of the Problem

We consider a model consisting of a homogeneous ocean of depth $h$, an elastic half-space and a sedimentary layer of thickness H between ocean and the half-space. The sedimentary layer is modeled as a homogeneous, transversely isotropic liquid-saturated porous solid layer and the half-space is taken as homogeneous, isotropic elastic medium.

## Basic Equations and their Solutions

The equations of motion for a porous solid (Biot; 1956 a, b) are

$$\tau_{ij,j} = \rho u_i + \rho_f W_i, \qquad (i, j = 1, 2, 3)$$

$$-(p_f)_{,i} = \rho_f u_i + \frac{c_i \rho_f}{\beta} W_i + F b_i \dot{W}_i, \quad (1)$$

where $\tau_{ij}$ is the stress tensor, $p_f$ is the pore fluid pressure, $\rho_f$ and $\rho$ are the mass densities of the fluid and the bulk porous material respectively. $\beta$ is the porosity of the porous medium. $\dot{W} = \beta(\dot{U} - \dot{u})$ is the relative velocity of the fluid with respect to the solid and $U$ and $u$ are the displacements of the liquid and the solid part of the porous medium.

Coefficients $b_i^{'} s$ are the friction parameters and are

$$b_i = \xi / k_i, \qquad (2)$$

where $\xi$ and $k_i$ are the viscosity and permeability of the pores fluid. Function $F(\kappa)$ is a frequency dependent viscosity factor, which is defined as

$$F(\kappa) = \frac{\kappa T(\kappa)}{4\{1 - 2/(\iota\kappa)\, T(\kappa)\}},$$

$$T(\kappa) = \frac{ber'(\kappa) + \iota\, bei'(\kappa)}{ber\,(\kappa) + \iota\, bei\,(\kappa)},$$

$$\kappa = \bar{a}\,(\omega \rho_f / \xi)^{1/2}, \qquad (3)$$

where $ber(\kappa)$ $bei(\kappa)$ are the real and imaginary parts of the Kelvin function

and for cylindrical pores, the permeability is given by (Biot, 1956 b)

$$k_i = (8/a^{-2})\,\delta_i, \qquad (4)$$

where $\bar{a}$ is the pore size and $\delta_i$ is a shape factor and its value is one for the circular cylindrical pores. The constants $c_i$ in equation (1) account for the added mass of the fluid associated with its motion relative to the solid. In the case of straight pores, the added mass is negligible and both the constants are unity.

The constitutive equations for a transversely isotropic porous medium (Biot, 1962 a) are given by

$$\tau_{xx} = 2B_1 e_{xx} + B_2(e_{xx}+e_{yy}) + B_3 e_{zz} + B_6\xi$$
$$\tau_{yy} = 2B_1 e_{yy} + B_2(e_{xx}+e_{yy}) + B_3 e_{zz} + B_6\xi$$
$$\tau_{zz} = B_4 e_{zz} + B_3(e_{xx}+e_{yy}) + B_7\xi,$$
$$\tau_{yz} = 2B_5 e_{yz},$$
$$\tau_{zx} = 2B_5 e_{zx},$$
$$\tau_{xy} = 2B_1 e_{xy},$$
$$p_f = B_6(e_{xx}+e_{yy}) + B_7 e_{zz} + B_8\xi, \qquad (5)$$

where $e_{xx}$, $e_{yy}$, etc. and $\tau_{xx}, \tau_{yy}$, etc. are the components of the strain tensor and stress tensor respectively., $\xi$ is the increment of the fluid content per unit volume which is defined as

$$\xi = div\,[\beta\,(u-U)]. \qquad (6)$$

There are eight material coefficients $B_1, B_2, ...., B_8$ in equation (5), which are to be evaluated by applying the method developed by Hashin and Rosen (1964) and by Christensen (1979) for evaluating the material coefficients of composite materials. The equations, which relate the coefficients $B_1, B_2, ...., B_8$ to the bulk modulus ($K_s$), shear modulus ($\mu_s$), Young's modulus ($E_s$), and Poisson's ratio ($\upsilon_s$) of the solid grain and the bulk modulus ($K_f$) of the pore fluid, and to the porosity ($\beta$) of the medium, are given by

$$B_1 = \mu_{12},$$
$$B_2 = K_{12} - \mu_{12},$$
$$B_3 = 2\upsilon_{13} K_{12},$$
$$B_4 = E_{33} + 4\upsilon_{31}^2 K_{12},$$
$$B_5 = 1/\mu_{13},$$
$$B_6 = -\frac{K_f\,(K_s + 4\mu_s/3)}{K_f + \mu_s + \beta(K_s + \mu_s/3 - K_f)},$$

$$B_7 = -K_f\,[1 + (1-\beta)\frac{2\upsilon_s\,(K_s+\mu_s/3) - K_f}{K_f + \mu_s + \beta(K_s + \mu_s/3 - K_f}]$$

$$B_8 = \frac{K_f\,[(K_s + 4\mu_s/3)\,\beta + \mu_s]}{K_f + \mu_s + \beta(K_s + \mu_s/3 - K_f)} \qquad (7a)$$

where

$$E_{33} = (1-\beta)E_s + \frac{4\beta(1-\beta)(1/2-\upsilon_s)^2}{(1-\beta)/K_f + \beta/(K_s+\mu_s/3) + 1/\mu_s}$$

$$\upsilon_{31} = (1-\beta)\upsilon_s + \beta/2 + \frac{\beta\,(1-\beta)\,(1/2-\upsilon_s)[1/(K_s+\mu_s/3) - 1/K_f]}{(1-\beta)/K_f + \beta/(K_s+\mu_s/3) + 1/\mu_s}$$

$$K_{12} = K_s + \mu_s/3 + \frac{\beta}{1/(K_f - K_s - \mu_s/3) + (1-\beta)/(K_s + 4\mu_s/3)}$$

$$\mu_{13} = (1-\beta)/\{(1+\beta)\}\mu_s.$$

The term $\mu_{12}$ is determined from the equation (Christensen, 1979)

$$A\,(\mu_{12}/\mu_s)^2 + 2B\,(\mu_{12}/\mu_s) + C = 0, \qquad (7b)$$

where

$$A = -3\beta(1-\beta)^2 - (\eta_s + \beta^3)(1+\beta\eta_s),$$

$$B = 3\beta(1-\beta)^2 + (1-\beta)(\eta_s - 1 + 2\beta^3)/2 - \frac{\beta}{2}(\eta_s + 1)(1-\beta^3)$$

$$C = -3\beta(1-\beta)^2 + (1-\beta)(1-\beta^3),$$

$$\eta_s = 3 - 4\upsilon_s.$$

Considering the two-dimensional wave motion in the $xz$ plane, the dynamical equations leads to

$$(2B_1 + B_2)\frac{\partial^2 u_x}{\partial x^2} + B_5\frac{\partial^2 u_x}{\partial z^2} + (B_3 + B_5)\frac{\partial^2 u_z}{\partial x \partial z} - B_6\frac{\partial^2 W_x}{\partial x^2} - B_6\frac{\partial^2 W_z}{\partial x \partial z} = \rho u_x + \rho_f W_x$$

$$(B_3 + B_5)\frac{\partial^2 u_x}{\partial x \partial z} + B_5\frac{\partial^2 u_z}{\partial x^2} + B_4\frac{\partial^2 u_z}{\partial z^2} - B_7\frac{\partial^2 W_x}{\partial x \partial z} - B_7\frac{\partial^2 W_z}{\partial z^2} = \rho u_z + \rho_f W_z$$

$$B_6\frac{\partial^2 u_x}{\partial x^2} + B_7\frac{\partial^2 u_z}{\partial x \partial z} - B_8\frac{\partial^2 W_x}{\partial x^2} - B_8\frac{\partial^2 W_z}{\partial x \partial z} = -\rho_f u_x - \frac{c_1 \rho_f}{\beta} W_x - F b_1 W_x$$

$$B_6\frac{\partial^2 u_x}{\partial x \partial z} + B_7\frac{\partial^2 u_z}{\partial z^2} - B_8\frac{\partial^2 W_x}{\partial x \partial z} - B_8\frac{\partial^2 W_z}{\partial z^2} = -\rho_f u_z - \frac{c_3 \rho_f}{\beta} W_z - F b_3 W_z \qquad (8)$$

We assume the plane wave solutions of equations (8) in the form

$$u_x = A_1 \exp[\iota\omega(t - x/c - qz)],$$
$$u_z = A_2 \exp[\iota\omega(t - x/c - qz)],$$
$$W_x = A_3 \exp[\iota\omega(t - x/c - qz)],$$
$$W_z = A_4 \exp[\iota\omega(t - x/c - qz)] \qquad (9)$$

Equation (8) and (9) leads to

$$\{\rho - B_5 q^2 - (2B_1 + B_2)/c^2\} A_1 - (B_3 + B_5)(q/c) A_2 + (\rho_f + B_6/c^2) A_3 + B_6 (q/c) A_4 = 0,$$

$$-(B_3 + B_5)(q/c) A_1 + \{\rho - B_4 q^2 - B_5/c^2\} A_2 + B_7 (q/c) A_3 + (\rho_f + B_7 q^2) A_4 = 0,$$

$$(B_6/c^2 + \rho_f) A_1 + B_7 (q/c) A_2 + \{-B_8/c^2 + c_1 \rho_f/\beta - \iota F b_1/\omega\} A_3 - B_8 (q/c) A_4 = 0,$$

$$B_6 (q/c) A_1 + (\rho_f + B_7 q^2) A_2 - B_8 (q/c) A_3 + \{-B_8 q^2 + c_3 \rho_f/\beta - \iota F b_3/\omega\} A_4 = 0. \qquad (10)$$

The above system of equation posses a non-trivial solution if $\det[a_{ij}] = 0$, (11)

where $a_{ij}$, the entries of symmetric matrix of order 4, are

$$a_{11} = \rho - (2B_1 + B_2)/c^2 - B_5 q^2,$$
$$a_{12} = a_{21} = -(B_3 + B_5)(q/c),$$
$$a_{13} = a_{31} = \rho_f + B_6/c^2,$$
$$a_{14} = a_{41} = B_6 (q/c),$$
$$a_{22} = \rho - B_5/c^2 - B_4 q^2,$$
$$a_{23} = a_{32} = B_7 (q/c),$$
$$a_{24} = a_{42} = \rho_f + B_7 q^2,$$
$$a_{33} = -B_8/c^2 + c_1 \rho_f/\beta - \iota F b_1/\omega,$$
$$a_{34} = a_{43} = -B_8 (q/c),$$
$$a_{44} = -B_8 q^2 + c_3 \rho_f/\beta - \iota F b_3/\omega.$$

The above equation is a cubic in $q^2$ and can be written as

$$T_0 q^6 + T_1 q^4 + T_2 q^2 + T_3 = 0, \qquad (12)$$

where

$$T_0 = c_{11} B_5 (B_7^2 - B_4 B_8),$$
$$T_1 = T_{11} + T_{12}/c^2,$$
$$T_2 = T_{21} + T_{22}/c^2 + T_{23}/c^4,$$
$$T_3 = T_{31} + T_{32}/c^2 + T_{33}/c^4 + T_{34}/c^6,$$

$$T_{11} = c_{11}(B_4 B_8 - B_7^2)\rho + c_{11}\rho B_5 B_8 + 2c_{11}\rho_f B_5 B_7 + c_{11} c_{33} B_4 B_5 + \rho_f^2 (B_7^2 - B_4 B_8),$$

$$T_{12} = c_{11}(2B_1 + B_2)(B_7^2 - B_4 B_8) - c_{33} B_4 B_5 B_8 + c_{33} B_5 B_7^2 + c_{11}(B_3^2 B_8 + 2B_3 B_5 B_8) - c_{11} B_6 (2B_3 B_7 + 2B_5 B_7 - B_4 B_6),$$

$$T_{21} = -c_{11}\rho^2 B_8 - 2c_{11}\rho\rho_f B_7 - c_{11} c_{33}\rho(B_4 + B_5) + c_{11}\rho_f^2 B_5 + c_{33}\rho_f^2 B_4 + \rho\rho_f^2 B_8 + 2\rho_f^3 B_7,$$

$$T_{22} = \rho(c_{11} B_5 B_8 + c_{33} B_4 B_8) + c_{11}\rho(2B_1 + B_2)B_8 + 2c_{11}\rho_f B_7(2B_1 + B_2) - c_{33}\rho B_7^2 + c_{11} c_{33} B_4(2B_1 + B_2) + c_{33}\rho B_5 B_8 - 4\rho_f^2 B_5 B_8 - 2\rho_f(B_3 + B_5)(c_{11} B_6 + c_{33} B_7) - 2\rho_f^2 B_3 B_8 - c_{11} c_{33}(B_3^2 + 2B_3 B_5) + 2c_{33}\rho_f B_4 B_6 + 2\rho_f^2 B_6 B_7 - c_{11}\rho B_6^2,$$

$$T_{23} = c_{33} B_7^2 (2B_1 + B_2) + c_{33}(B_3 + B_5)\{B_3 B_8 - 2B_6 B_7) - (c_{11} B_5 + c_{33} B_4)\{(2B_1 + B_2)B_8 - B_6^2\}$$

$$T_{31} = \rho^2 c_{11} c_{33} - \rho\rho_f^2 (c_{11} + c_{33}) + \rho_f^4,$$

$$T_{32} = \rho c_{33}\{c_{11}(B_5 + 2B_1 + B_2) - \rho B_8 - 2\rho_f B_6\} + \rho_f^2\{\rho B_8 + 2\rho_f B_6 + c_{11}(2B_1 + B_2) + c_{33} B_5\},$$

$$T_{33} = c_{33}\rho B_8(B_5 + 2B_1 + B_2) + c_{11} c_{33} B_5(2B_1 + B_2) + \rho_f^2\{B_6^2 - B_8(2B_1 + B_2)\} + c_{33} B_6\{2\rho_f B_5 - \rho B_6\},$$

$$T_{34} = c_{33} B_5\{B_6^2 - B_8(2B_1 + B_2)\};$$

$$c_{11} = \frac{c_1\rho_f}{\beta} - \imath\frac{F b_1}{\omega},$$

$$c_{33} = \frac{c_3\rho_f}{\beta} - \imath\frac{F b_3}{\omega}. \qquad (13)$$

The roots of the equation (12) are, in general, complex. We denote these roots by $q(n), \quad n = 1, 2, ...., 6$. Three roots with positive real parts will correspond to the waves traveling in the positive z-direction (downgoing waves) and other three roots with negative real parts will correspond to the waves traveling in the negative z-direction (upgoing waves). We order the six roots $q(n), \quad n = 1, 2, ...., 6$ such that $q(1), q(2), q(3)$ correspond to upgoing waves and $q(6), q(5), q(4)$ correspond to the downgoing waves. These are quasi $P_I$, quasi $P_{II}$ and quasi $SV$ waves respectively. Substituting $q(n)$ into equation (10), the wave amplitudes $A_1, A_2, A_3,$ and $A_4$ can be obtained. We denote the corresponding normalized eigen vectors by $A_i(n), (i = 1, 2, 3, 4;\ n = 1, 2, ....., 6)$. These are given by

$$A_1(n) = X_1(n)/X(n), \quad A_2(n) = X_2(n)/X(n),$$
$$A_3(n) = X_3(n)/X(n), \quad A_4(n) = X_4(n)/X(n),$$

where

$$X_1(n) = q^4(n) c_{11}(B_4 B_8 - B_7^2) + q^2(n)\{B_8(B_5 c_{11} + B_4 c_{33})/c^2 - c_{11} B_8\rho - 2c_{11}\rho_f B_7 + c_{33}(c_{11} B_4 - B_7^2/c_{33})\} + \{c_{33}\frac{B_8}{c^2}(B_5/c^2 - \rho) - c_{11} c_{33}(B_5/c^2 - \rho) + \rho_f^2(B_8/c^2 + c_{11})\},$$

$$X_2(n) = q^3(n)\frac{c_{11}}{c}\{B_6 B_7 - B_3 B_8 - B_5 B_8\} + \frac{q(n)}{c}\{[B_7(\rho_f + B_6/c^2) - B_8(B_3 + B_5)/c^2]c_{33} + c_{11}\rho_f B_6 + \rho_f^2 B_8 + c_{11} c_{33}(B_3 + B_5)\},$$

$$X_3(n) = q^4(n)\rho_f\,(B_7{}^2 - B_4 B_8) + q^2(n)\{c_{33} B_4(\rho_f + B_6/c^2) - (c_{33} B_7 + \rho_f B_8)(B_3 + B_5)/c^2 + \rho_f B_8(\rho - B_5/c^2) + \rho_f B_7(\rho_f + B_6/c^2) + B_7 \rho_f{}^2\} + (\rho_f + B_6/c^2)\{\rho_f{}^2 - c_{33}(\rho - B_5/c^2)\},$$

$$X_4(n) = \frac{q^3(n)}{c}\{c_{11}(B_4B_6 - B_3B_7 - B_5B_7) + (B_4B_8 - B_7{}^2)\rho_f\} + \frac{q(n)}{c}\{-c_{11}\rho_f(B_3 + B_5) - c_{11}(\rho - B_5/c^2)B_6 + \rho_f B_8(B_3 + 2B_5)/c^2 - \rho\rho_f B_8 - \rho_f B_7(\rho_f + B_6/c^2)\},$$

and

$$X(n) = \sqrt{(X_1(n))^2 + (X_2(n))^2 + (X_3(n))^2 + (X_4(n))^2}$$

The constitutive equations for homogeneous, isotropic elastic solid are

$\sigma^*_{xx} = \lambda\theta + 2\mu\varepsilon_{xx}$,

$\sigma^*_{zz} = \lambda\theta + 2\mu\varepsilon_{zz}$,

$\sigma^*_{zx} = 2\mu\varepsilon_{zx}$,

where $\varepsilon_{xx}, \varepsilon_{zx}$ etc. are the components of strain tensor; $\sigma^*_{xx}, \sigma^*_{zx}$ etc. are the components of stress tensor; and $\theta$ is the dilatation. Substituting these equations in the equations of motion

$\sigma^*_{ij,j} = \rho^* u^*_i$,

The equation of motion in the elastic solid half space is

$$(\lambda + 2\mu)\frac{\partial^2 u^*}{\partial x^2} + \mu\frac{\partial^2 u^*}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w^*}{\partial x \partial z} = \rho^* u^*$$

$$(\lambda + \mu)\frac{\partial^2 u^*}{\partial x \partial z} + \mu\frac{\partial^2 w^*}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 w^*}{\partial z^2} = \rho^* w^* \qquad (14)$$

where $u^*$ and $w^*$ are tangential and normal displacements, $\lambda$ and $\mu$ are Lame's constants and $\rho^*$ is the density. When we consider

$u^* = A_1^* \exp[\iota\omega(t - x/c - q^* z)]$,

$w^* = A_2^* \exp[\iota\omega(t - x/c - q^* z)]$, (15)

then equation (14) becomes

$$[(\lambda + 2\mu)/c^2 + \mu q^2 - \rho^*]A_1^* + (\lambda + \mu)\frac{q^*}{c}A_2^* = 0$$

$$(\lambda + \mu)\frac{q^*}{c}A_1^* + [\mu/c^2 + (\lambda + 2\mu)q^2 - \rho^*]A_2^* = 0 \qquad (16)$$

This leads to a quadratic in $q^2$, which is

$A^* q^4 + B^* q^2 + C^* = 0$, (17)

where

$A^* = (\lambda + 2\mu)\mu$,

$B^* = 2\mu(\lambda + 2\mu)/c^2 - \rho^*(\lambda + 3\mu)$,

$C^* = \mu(\lambda + 2\mu)/c^4 - \rho^*(\lambda + 3\mu)/c^2 + \rho^2$

We denote the four roots of the equation (17) as $q^*(n)$, n=1,…,4. Out of the four roots $q^*(n)$, (n=1,2,…,4) we assume that $q^*(1)$ and $q^*(2)$ correspond to $P$ and $SV$ waves propagating in the negative z-direction, and $q^*(3)$ and $q^*(4)$ correspond to $SV$ and $P$ wave propagating in positive z direction. Substituting the values of $q^*(n)$ in the equation (16) and solving for $A_1^*$ and $A_2^*$, we obtain

$$A_1^* = K_1^*(n)/K^*(n),\ \ A_2^* = K_2^*(n)/K^*(n), \qquad (18)$$

where

$K_1^*(n) = \mu/c^2 + (\lambda + 2\mu)\{q^*(n)\}^2 - \rho^*$,

$K_2^*(n) = -(\lambda + \mu) q^* / c$,

and

$K^*(n) = \sqrt{\{K_1^*(n)\}^2 + \{K_2^*(n)\}^2}$,

n=1, 2, 3, 4.

Similarly, writing solutions

$u' = A_1' \exp[\iota\omega(t - x/c - q'z)]$,

$w' = A_2' \exp[\iota\omega(t - x/c - q'z)]$, (19)

in the liquid layer, we obtain

$q'(n) = \pm \frac{1}{c}\sqrt{(c^2 - \alpha_0^2)/\alpha_0^2}$, (n=1, 2) (20)

and their corresponding normalized eigen vectors $A_1'(n)$ and $A_2'(n)$.

**Reflection and Refraction of plane waves**

We assume that incident wave ($P$ or $SV$ wave) travels in the elastic basement and impinges on the surface of sediment layer at an angle $\theta_0$. In the sediment layer, the waves are reflected many times by the ocean sediment interface and the sediment-basement interface. In the liquid layer, the multiple reflections at the free surface and the ocean-sediment interface create the wave field composed of upgoing and downgoing compressional waves.

The displacement components associated with the incident and reflected waves in the elastic half-space can be written as

$$u^* = \sum_{n=1}^{2} A_1^*(n) \exp[\iota\omega(t - x/c - q^*(n) z)] + \sum_{n=3}^{4} A_1^*(n) f^*(n) \exp[\iota\omega(t - x/c - q^*(n) z)],$$

$$w^* = \sum_{n=1}^{2} A_2^*(n) \exp[\iota\omega(t - x/c - q^*(n) z)] + \sum_{n=3}^{4} A_2^*(n) f^*(n) \exp[\iota\omega(t - x/c - q^*(n) z)], \quad (21)$$

where $f^*(3)$ and $f^*(4)$ are the relative wave amplitudes of reflected $SV$ and reflected $P$ wave respectively. For incident $P$ wave,

$A_1^*(2) = A_2^*(2) = 0$,

and for incident $SV$ wave,

$A_1^*(1) = A_2^*(1) = 0$.

In the liquid layer, the displacement components corresponding to upward and downward traveling $P$ wave can be expressed as

$$u' = \sum_{n=1}^{2} e'(n) A_1'(n) \exp(\iota\omega(t - x/c - q'(n) z)]$$

$$w' = \sum_{n=1}^{2} e'(n) A_2'(n) \exp(\iota\omega(t - x/c - q'(n) z)] \quad (22)$$

where $e'(1)$ and $e'(2)$ are the relative amplitudes of upgoing and downgoing $P$ wave in the liquid layer.

The displacement components associated with upgoing and downgoing quasi body waves in the transversely isotropic porous layer can be written as

$$u_x = \sum_{n=1}^{6} g(n) A_1(n) \exp[\iota\omega(t - x/c - q(n) z)]$$

$$u_z = \sum_{n=1}^{6} g(n) A_2(n) \exp[\iota\omega(t - x/c - q(n) z)]$$

$$W_x = \sum_{n=1}^{6} g(n) A_3(n) \exp[\iota\omega(t - x/c - q(n) z)]$$

$$W_z = \sum_{n=1}^{6} g(n) A_4(n) \exp[\iota\omega(t - x/c - q(n) z)] \quad (23)$$

where $g(1)$, $g(6)$; $g(2)$, $g(5)$; $g(3)$, $g(4)$ are relative amplitudes of upgoing and downgoing transmitted quasi $P_I; P_{II}$ and $SV$ waves respectively.

The wave number $k$ is given by

$$k = \frac{\omega}{c} = \frac{\omega}{V_0}\sin\theta_0, \qquad (24)$$

where $\theta_0$ is the angle of incidence and $V_0$ is the velocity of incident wave.

## Boundary Conditions

The boundary conditions at the free surface ($z = -h$) and porous fluid interface ($z = 0$) are

i) $p' = 0$, at $z = -h$,

ii) $\tau_{zz} = -p' = -\lambda_0\left(\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z}\right)$, at $z = 0$,

iii) $\tau_{zx} = 0$, at $z = 0$,

iv) $p_f - p' = \phi\, \dot{W}_z$, at $z = 0$,

v) $u_z + W_z = w'$ at $z = 0$. (25a)

Vashishth et al. (1991) discussed boundary conditions at an imperfect interface. Treating the boundary between porous layer and elastic half space as loose boundary, the boundary conditions there at i.e. at $z = H$ are

vi) $\tau_{zz} = \sigma^*_{zz}$,

vii) $\tau_{zx} = \sigma^*_{zx}$,

viii) $u_z = w^*$,

ix) $\dot{W}_z = 0$,

x) $\sigma^*_{zx} = \iota\omega\left(\frac{\psi}{1-\psi}\right)\frac{\mu}{V_0}(u^* - u_x)$, (25b)

where $p'$ is the liquid pressure, $\lambda_0$ is Lame constant in the liquid layer. $\phi$ is a kind of surface flow impedance and its value is zero when the pores are open and $\phi$ is infinite for the seabed pores. $\psi$ is the bonding parameter whose value lies between one and zero and is one for welded interface and is zero for smooth interface.

Substituting the values of the displacement components from (21) to (23) into the above boundary conditions and making use of (5) and (15), we obtain a system of linear non-homogeneous equations in ten unknowns and these can be expressed as

$$\mathbf{X}\,\mathbf{Z} = \mathbf{Y}, \qquad (26)$$

where $\mathbf{X}$ is a square matrix of order 10 and $\mathbf{Y}$ and $\mathbf{Z}$ are column matrices. The elements of the matrix $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ are given in the Appendix A. The system of equations (26) can be solved to obtain the ten unknowns $z_j\,(j = 1, 2, ...., 10)$. These are the amplitude ratios of reflected and transmitted waves.

## Numerical Computation and Discussion of Results:

For numerical calculations we take a particular model and its parameters are given below. For the water layer, the values of the density, elastic parameter and compressional wave velocity are taken as

$\rho' = 1.025\, gm/cm^3, \lambda_0 = 1.92\, dyne/cm^2,$

$\alpha_0 = 1.37\times10^5\ cm/\sec.$

The physical properties of the sediment, which is taken as water saturated medium sand, are given (Yamamoto, 1983 b) by

$K_s = 3.0\times10^9\, dyne/cm^2, \mu_s = 1.0\times10^9\, dyne/cm^2,$

$\upsilon_s = 0.35,\ E_s = 2.7\times10^9\, dyne/cm^2,$

$\rho_s = 2.65\, gm/cm^3,\ \rho_f = 1.025\, gm/cm^3,$

$K_f = 1.92\times10^{10}\, dyne/cm^2, \beta = .45,$

$\xi = 1.025\times10^{-2}\, dyne-\sec/cm^2,$

$c_1 = c_3 = 1.25, k_1 = k_3 = 1.0\times10^{-6} cm^2,$

The parameters of the Bedrock (elastic half-space) are
$\rho^* = 2.6\, gm/cm^3, \lambda = 2.5\times10^{11}\, dyne/cm^2,$
$\mu = 3.75\times10^{11}\, dyne/cm^2.$

Using these values of the parameters for liquid layer, sedimentary layer and for the elastic half-space, the equation (26) is solved numerically to determine the amplitude ratios of different reflected and refracted waves.

Fig. 1 and 2, show the amplitude ratios of reflected $P$ and $SV$ waves for all angle of incidence and for different degrees of bonding of the interface between elastic half-space and porous layer. It is evident from the figures that mode conversion does not take place. It is also clear that the effect of looseness is more predominant for the $SV$ wave as compared to $P$ wave. At the normal incidence and for small incident angles, the looseness of the interface affects the reflection coefficient of shear wave significantly. Fig. 3 to 8 shows the amplitude ratios of the three upward traveling and three downward traveling transmitted waves. It is evident that the bondedness of the interface influences the reflection-refraction phenomena except for angles near the grazing incidence, as excepted. The amplitude ratios of upgoing transmitted $P_I$ wave and $SV$ wave increase with the bondedness of the sediment-basement interface while those of downgoing waves decrease as the bondedness increases. For the $P_{II}$ wave, the behavior is different and the amplitude ratios for both upgoing and downgoing waves increase with the bonding parameters (Fig. 6 & 7). It is also noted that the amplitude ratios of upgoing $P_I$ and $SV$ waves are greater than the corresponding downward traveling waves. The less energy goes into the slow compressional waves in comparison to the other waves. The amplitudes of the compressional waves propagating in the positive and negative z-direction in the liquid layer are shown in the Fig. 9. The amplitudes of upward and downward going $P$ waves in the liquid layer are exactly same (Fig. 9). The bonding parameter ($\psi$) of the porous layer-basement interface show a little effect on the wave in the water.

The critical angle for this wave is at about $37^\circ$. Amplitude ratio of reflected $SV$ wave decreases with the angle of incidence as the angle of incidence varies from $0^\circ$ to $34^\circ$ and then starts increasing as the angle of incidence approaches critical angle. It is also clear from this figure (11) that the amplitude ratio decreases with the decrease in bonding parameter. It is clear from Figs. 12 and 13 that the amplitude of downward going $P_I$ wave is more than that of upgoing wave in the sediment. The amplitude of both the waves increases with the angle of incidence. The amplitude of upgoing $P_I$ wave decreases as the bonding parameter varies from 1.0 to 0.5 but it increases if the looseness of the boundary increases further and attains the maximum for an ideally smooth interface. However, the amplitude ratio of downward going $P_I$ wave shows a continuous decrement with the looseness of the bonding interface (Fig. 13). The amplitude of upgoing $P_{II}$ wave is slightly greater than of corresponding downgoing wave (Figs. 14 & 15). It is evident from the Figs 16 and 17 that the amplitude of downward going shear wave is greater than that of upgoing

shear wave. The variation of the amplitude ratios of $SV$ waves is similar to that of upgoing and downgoing $P_I$ wave. The amplitudes of upgoing and downgoing compressional waves in the liquid layer are same (Fig. 18) and increases with the incident angle. The effect of imperfect bonding of sediment-basement interface on the amplitude of $P$ wave in the homogeneous ocean is not significant.

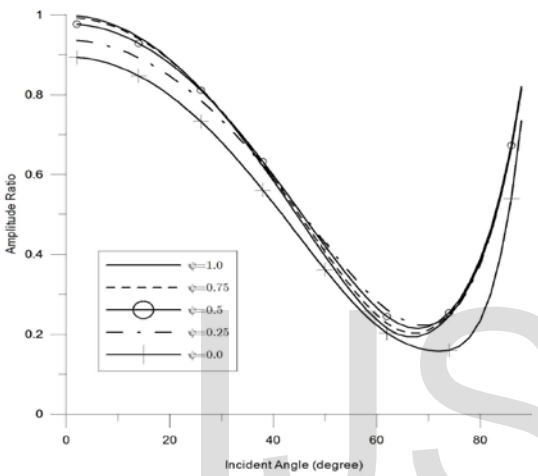


Fig. 1 The reflection coefficient of P wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

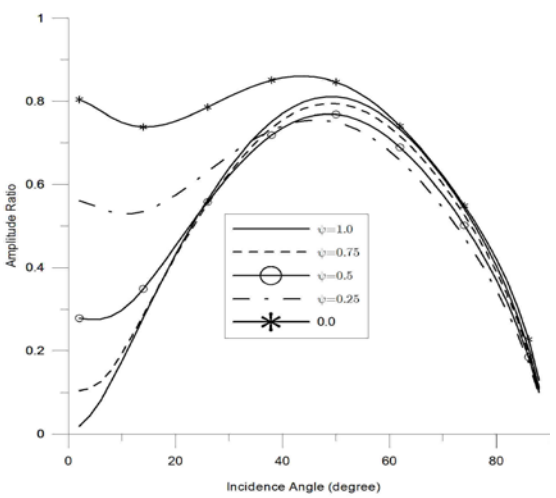


Fig. 2 The reflection coefficient of shear wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

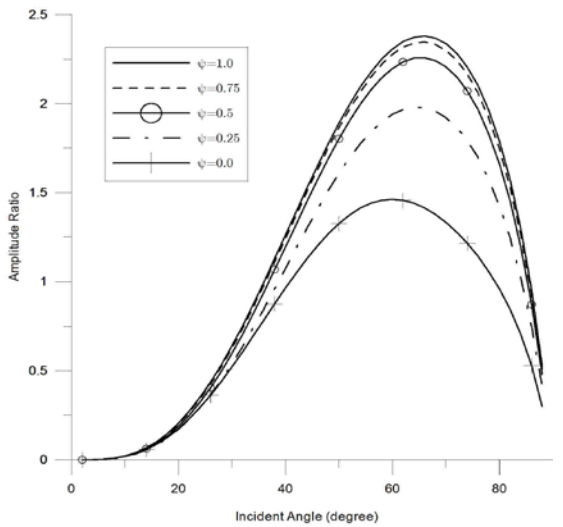


Fig. 3 The transmission coefficient ofupgoing $P_I$ wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

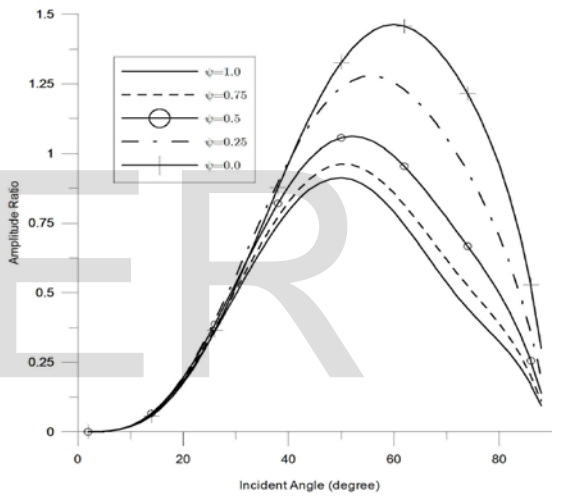


Fig. 4 The transmission coefficient of downgoing $P_I$ wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

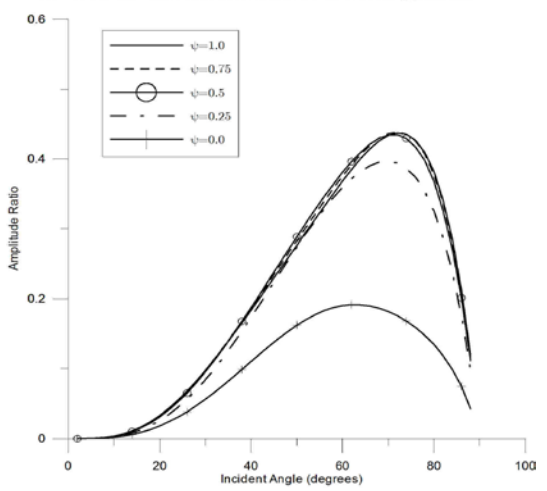


Fig. 5 The transmission coefficient of upgoing $P_s$ wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

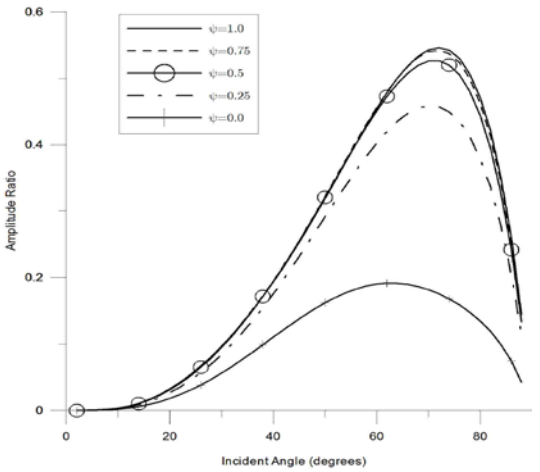


Fig. 6 The transmission coefficient of downgoing $P_s$ wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

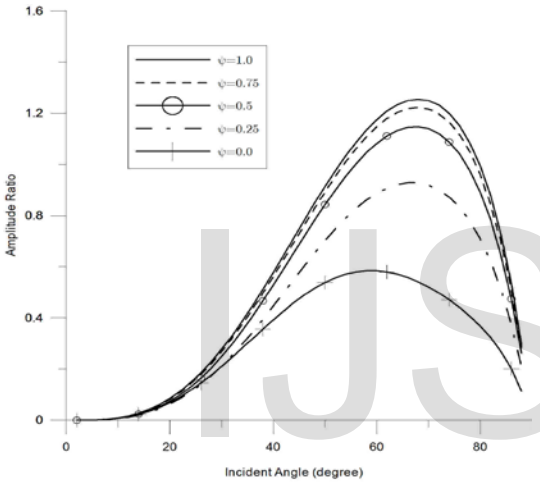


Fig. 7 The transmission coefficient of upgoing SV wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

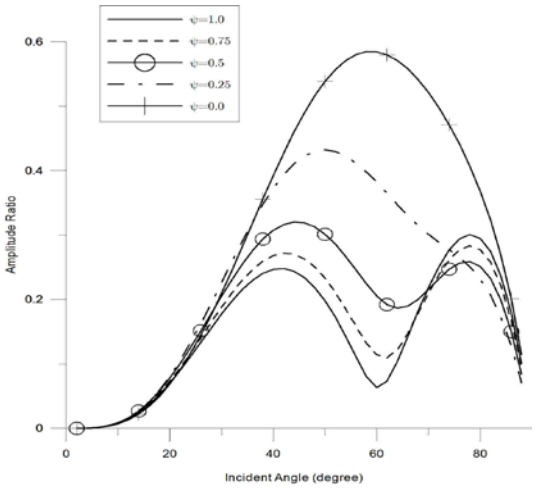


Fig. 8 The transmission coefficient of SV wave at incident angles (0-90) of P wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

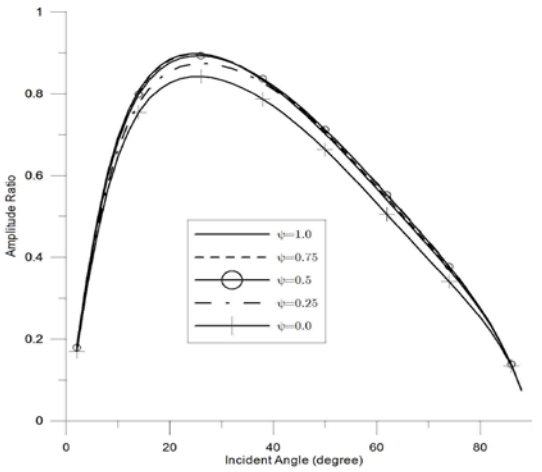


Fig. 9 The transmission coefficient of upgoing and downgoing compressional waves in liquid layer at incident angles (0-90) for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

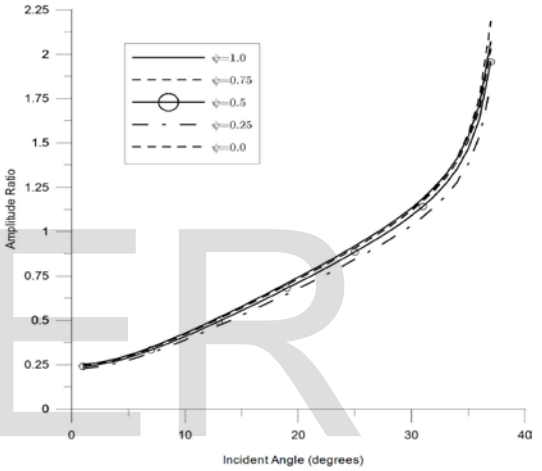


Fig. 10 The reflection coefficient of compressional wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

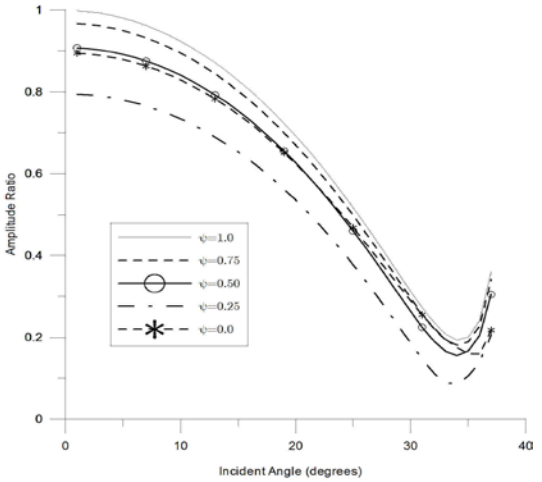


Fig. 11 The reflection coefficient of shear wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

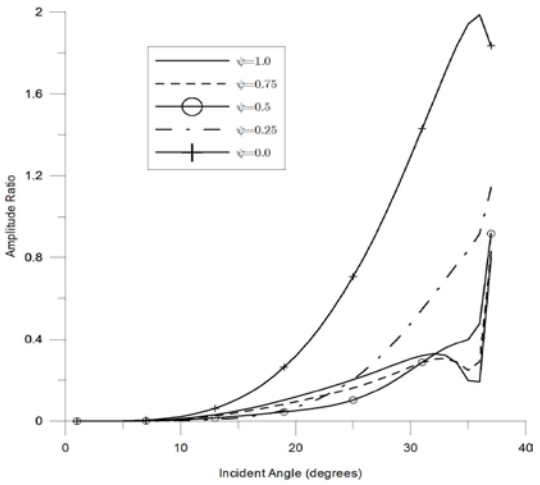


Fig. 12 The refraction coefficient of upgoing fast compressional waveversus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0of bonding parameter

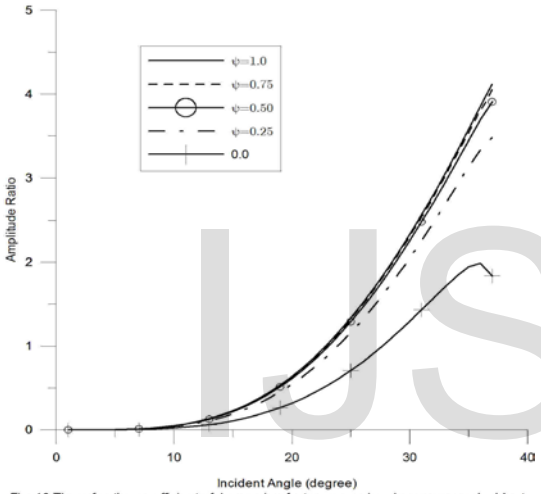


Fig. 13 The refraction coefficient of downgoing fast compressional wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

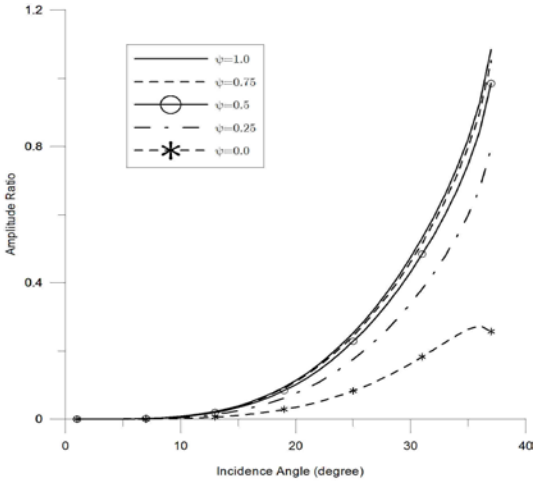


Fig. 14 The refraction coefficient of downgoing slow compressional wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

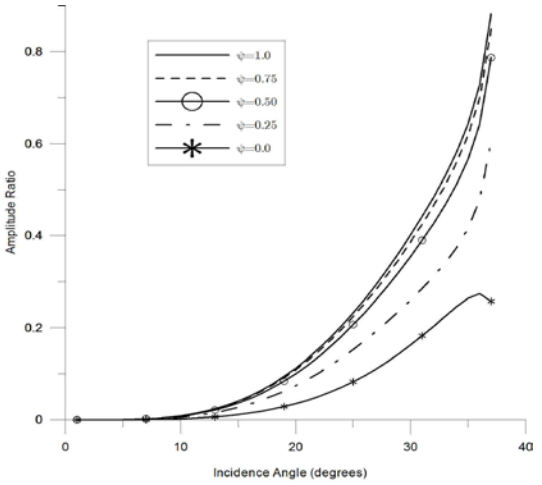


Fig. 15 The refraction coefficient of downgoing slow compressional wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

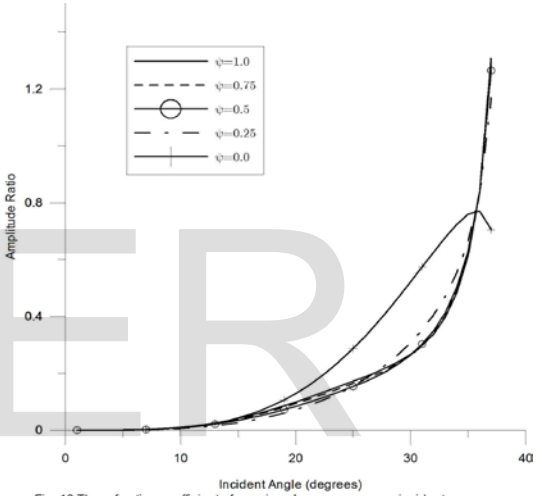


Fig. 16 The refraction coefficient of upgoing shear wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

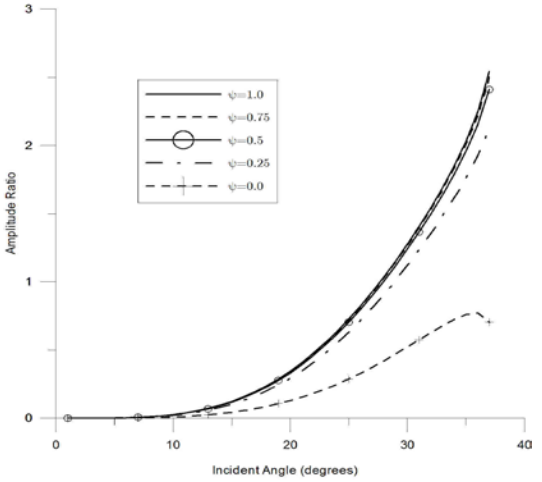


Fig. 17 The refraction coefficient of downgoing shear wave versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

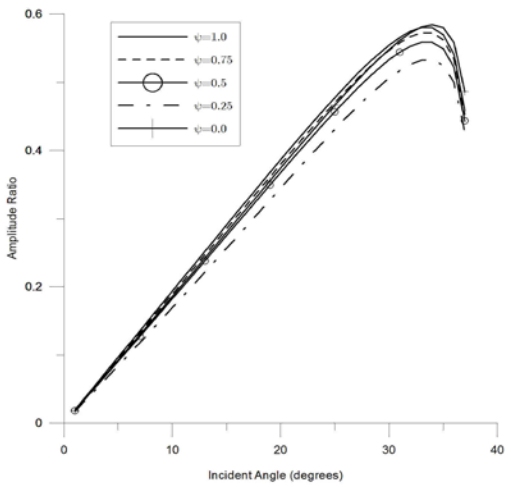


Fig. 18 The refraction coefficient of upgoing and downgoing compressional waves in liquid layer versus incident angle of SV wave for the values 1.0, 0.75, 0.5, 0.25 and 0.0 of bonding parameter

## Conclusion:

A mathematical model for a sedimentary layer underlying a homogeneous ocean and overlaying an elastic basement has been proposed to study the reflection-refraction phenomena. To make the study more realistic, the loss of energy due to the relative motion of the pore-water to the sediment frame has been taken into account and the sedimentary layer was treated as a transversely isotropic porous solid saturated with water. The partitioning of energy at the sedimentary layer-basement interface is greatly influenced by the imperfectness of interface. The sediment-basement interface is assumed to be a loosely bonded interface and the results for welded and smooth contact have been obtained as particular cases. Amplitude ratios of all the reflected and transmitted waves have been computed. The effect of the loose boundaries on the amplitude ratios has been studied. The calculated results reveal that the sediment beds and the ocean have some influence on the reflection of seismic waves at the sediment-basement interface.

## Appendix A:

$$x_{11} = x_{12} = ..... = x_{18} = 0\,,$$

$$x_{1k} = \{A_1'(n)/c + A_2'(n)\,q'(n)\}\exp[\iota\,\omega\,q'(n)\,h],$$
$$(n = 1, 2;\ k = n + 8)$$

$$x_{21} = x_{22} = 0\,,$$

$$x_{2j} = B_4\,A_2(n)\,q(n) + B_3\,A_1(n)/c -$$
$$B_7\,[\,A_3(n)/c + A_4(n)\,q(n)\,],$$
$$(n = 1, 2, .., 6;\ j = n + 2)$$

$$x_{2k} = -\lambda_0\,[\,A_1'(n)/c + A_2'(n)\,q'(n)\,], (n = 1, 2; k = n + 8)$$

$$x_{31} = x_{32} = 0\,,$$

$$x_{3j} = A_1(n)\,q(n) + A_2(n)/c,\ (n = 1, 2, ...., 6;\ \ j = n + 2)$$

$$x_{3k} = 0, \qquad (k = 9,\ 10)$$

$$x_{41} = x_{42} = 0\,,$$

$$x_{4j} = B_6\,A_1(n)/c + B_7\,A_2(n)\,q(n) + \phi\,A_4(n) -$$
$$B_8\,[\,A_3(n)/c + A_4(n)\,q(n)\,],$$
$$(n = 1, 2, ..., 6;\ j = n + 2)$$

$$x_{4k} = \lambda_0\,[\,A_1'(n)/c + A_2'(n)\,q'(n)\,], (n = 1, 2;\ k = n + 8)$$

$$x_{51} = x_{52} = 0\,,$$

$$x_{5j} = A_2(n) + A_4(n), \quad (n = 1, 2, ..., 6;\ j = n + 2)$$

$$x_{5k} = -A_2'(n), \qquad (n = 1, 2;\ k = n + 8)$$

$$x_{6i} = [\lambda\,A_1^*(n)/c + (\lambda + 2\mu)\,A_2^*(n)\,q^*(n)\,]\exp[-\iota\omega\,q^*(n)\,H]$$
$$(n = 3, 4;\ i = n - 2)$$

$$x_{6j} = -[B_4 A_2(n)\,q(n) + B_3 A_1(n)/c -$$
$$B_7\{A_3(n)/c + A_4(n)\,q(n)\}]\ \exp[-\iota\,\omega\,q(n)\,H],$$
$$(n = 1, 2, ..., 6;\ j = n + 2)$$

$$x_{6k} = 0, \qquad (k = 9, 10)$$

$$x_{7i} = \mu\{A_1^*(n)\,q^*(n) + A_2^*(n)/c\}\exp[-\iota\,\omega\,q(n)\,H],$$
$$(n = 3, 4;\ i = n - 2)$$

$$x_{7j} = -B_5\,\{A_1(n)\,q(n) + A_2(n)/c\}\exp[-\iota\,\omega\,q^*(n)\,H],$$
$$(n = 1, 2, ..., 6;\ j = n + 2)$$

$$x_{7k} = 0, \qquad (k = 9, 10)$$

$$x_{8i} = A_2^*(n)\exp[-\iota\,\omega\,q^*(n)\,H], \qquad (n = 3, 4;\ i = n - 2)$$

$$x_{8j} = -A_2(n)\exp[-\iota\,\omega\,q^*(n)\,H],\ \ (n = 1, 2, ..., 6;\ j = n + 2)$$

$$x_{8k} = 0, \qquad (k = 9, 10)$$

$$x_{9k} = 0, \quad (k = 1,2,9,10)$$

$$x_{9j} = A_4(n)\ \exp[-\iota\,\omega\, q(n)\, H], \quad (n = 1,2,...,6;\ j = n+2)$$

$$x_{10i} = [\psi A_1^*(n) + (1-\psi) V_0 \{ A_1^*(n) q^*(n) + A_2^*(n)/c \}] \exp[-\iota\,\omega\, q^*(n)\, H],$$

$$(n = 3,4;\ i = n-2)$$

$$x_{10j} = -A_1(n)\ \exp[-\iota\,\omega\, q(n)\, H], \quad (n = 1,2,...,6;\ j = n+2)$$

$$x_{10k} = 0, \quad (k = 9,10)$$

The elements of column matrix **Z** are given by

$$z_1 = f^*(3),\ z_2 = f^*(4),\ z_j = g(j-1),$$

$$(j = 3,4,...,8), z_9 = e'(1),\ z_{10} = e'(2).$$

The elements of the matrix **Y** are

$$y_1 = y_2 = ..... = y_5 = y_9 = 0,$$

$$y_6 = -\{ \lambda A_1^*(n)/c + (\lambda + 2\mu) A_2^*(n) q^*(n) \} \exp[-\iota\,\omega\, q^*(n)\ H]$$

$$y_7 = -\mu \{ A_1^*(n) q^*(n) + A_2^*(n)/c \} \exp[-\iota\,\omega\, q^*(n)\ H]$$

$$y_8 = -A_2^*(n)\ \exp[-\iota\,\omega\, q^*(n)\ H],$$

$$y_{10} = -[\psi A_1^*(n) + (1-\psi) V_0 \{ A_1^*(n) q^*(n) + A_2^*(n)/c \}] \exp[-\iota\,\omega\, q^*(n)\ H]$$

where n=1 for incident $P$ wave and n=2 for incident $SV$ wave.

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